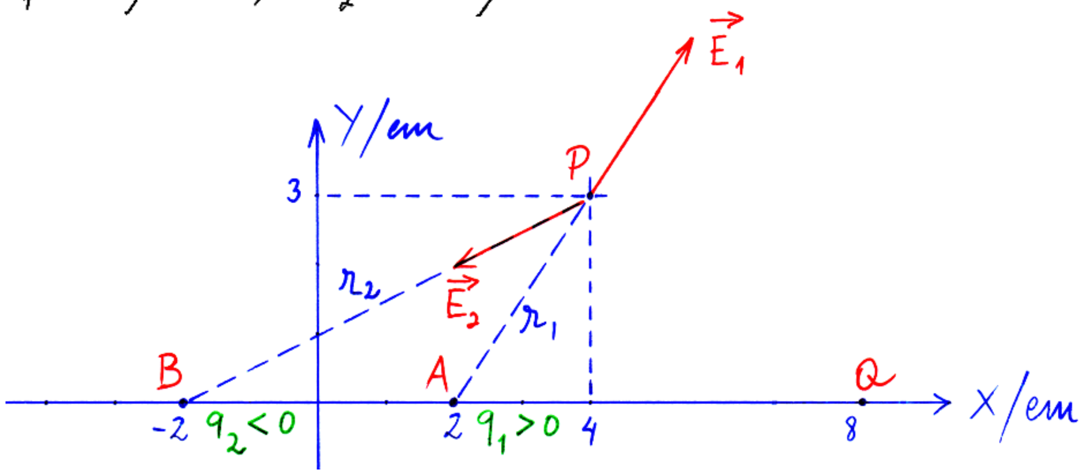
1. $q_1 = 3\mu C$, $q_2 = -6\mu C$



$$\hat{r}_1 = \frac{\overrightarrow{AP}}{\|\overrightarrow{AP}\|} = \frac{(4,3)-(2,0)}{\|(2,3)\|} = \frac{(2,3)}{\sqrt{13}} = \frac{1}{\sqrt{13}}(2\hat{i} + 3\hat{j})$$

$$\hat{r}_2 = \frac{\overrightarrow{BP}}{\|\overrightarrow{BP}\|} = \frac{(4,3)-(-2,0)}{\|(6,3)\|} = \frac{(6,3)}{\sqrt{45}} = \frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$$

1.1 $$\vec{E} = \vec{E}_1 + \vec{E}_2 = K\frac{q_1}{r_1^2}\hat{r}_1 + K\frac{q_2}{r_2^2}\hat{r}_2 =$$

$$= K\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2\right) =$$

$$= 9\times10^9 \times \left(\frac{3}{(\sqrt{13})^2}\cdot\frac{(2\hat{i}+3\hat{j})}{\sqrt{13}} + \frac{-6}{(\sqrt{45})^2}\cdot\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})\right)\times\frac{1\times10^{-6}}{(1\times10^{-2})^2}$$

$$= 9\times10^7\times\left[\left(\frac{6}{13\sqrt{13}} - \frac{12}{45\sqrt{5}}\right)\hat{i} + \left(\frac{9}{13\sqrt{13}} - \frac{6}{45\sqrt{5}}\right)\hat{j}\right] =$$

$$\approx \underline{787570\,\hat{i} + 11914481\,\hat{j}} \quad (N/C)$$

1.2

$$V_P = V_1 + V_2 = K\frac{q_1}{r_1} + k\frac{q_2}{r_2} =$$

$$= K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) =$$

$$= 9\times10^9 \times \left(\frac{3}{\sqrt{13}} + \frac{-6}{\sqrt{45}}\right) \times \frac{10^{-6}}{10^{-2}} =$$

$$= \underline{\underline{-56139\ V}}$$

1.3

$$\vec{F} = q_3\vec{E} = 2\times10^{-9} \times (787570\,\hat{i} + 11914481\,\hat{j})$$

$$= \underline{\underline{1{,}575\times10^{-3}\,\hat{i} + 2{,}383\times10^{-2}\,\hat{j}\ (N)}}$$

$$\underline{\underline{\|\vec{F}\| = 2{,}39\times10^{-2}\ N}}$$

1.4 Uma vez que o campo eléctrico é conservativo o trabalho não depende do caminho escolhido mas apenas dos pontos inicial e final.

$$W_{P\to Q}(\vec{F}_{ele}) = -\Delta E_p = -q\,\Delta V = q(V_P - V_Q)$$

$$V_Q = V_{1Q} + V_{2Q} = K\left(\frac{q_1}{r_{1Q}} + \frac{q_2}{r_{2Q}}\right) =$$

$$= 9\times10^9 \times \left(\frac{3}{6} + \frac{-6}{10}\right) \times \frac{10^{-6}}{10^{-2}} = -90000\ V$$

$$\therefore\ W_{P\to Q}(\vec{F}_{ele}) = 2\times10^{-6}\times(-56139 + 90000) = 67{,}7\ mJ$$

$$\therefore \; W_{P \to Q}(\vec{F}_{ext}) = \underline{\underline{-67{,}7 \text{ mJ}}}$$

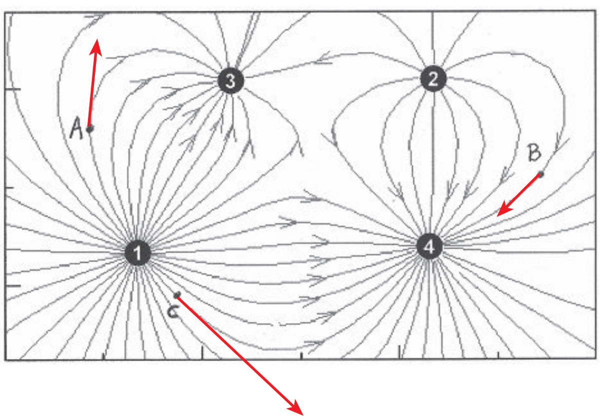
2.1 As linhas de campo eléctrico saem das cargas positivas e entram nas negativas e o seu número é proporcional ao valor da carga da qual saem ou entram, donde:

$$q_1 > 0 \, , \; q_2 > 0 \, , \; q_3 < 0 \, , \; q_4 < 0$$

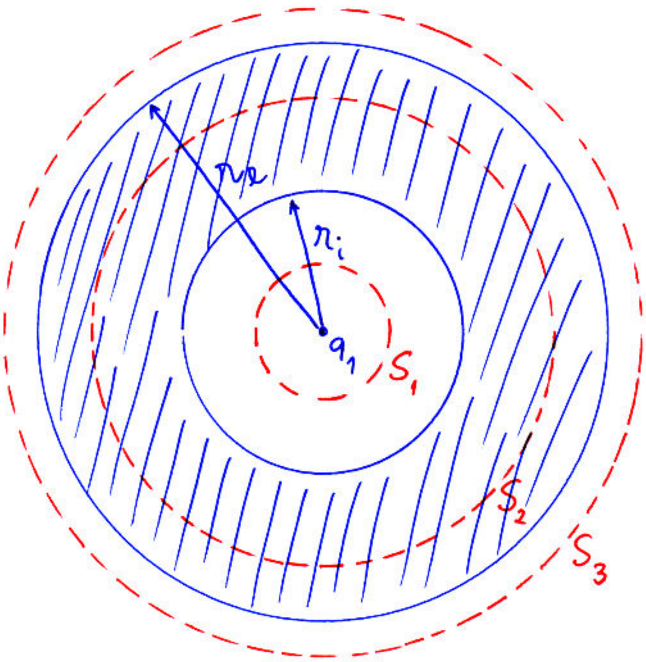
$$|q_2| < |q_3| < |q_4| < |q_1|$$

2.2 O vector campo eléctrico será tangente, em cada ponto, à linha de campo respectiva, apontando no sentido definido. A sua intensidade será proporcional à densidade de linhas de campo na região circundante:

$$\|\vec{E}_B\| < \|\vec{E}_A\| < \|\vec{E}_C\|$$



3



$$\rho = \frac{q_2}{V_2}$$

$$V_2 = \frac{4}{3}\pi(r_e^3 - r_i^3)$$

$q_1 = 6\ \mathrm{nC}$

$q_2 = -4\ \mathrm{nC}$

$r_i = 1\ \mathrm{m}$

$r_e = 2\ \mathrm{m}$

S_1 : $\boxed{r < r_i = 1\ \mathrm{m}}$

$$\phi_E = E \cdot S = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow E \cdot 4\pi r^2 = \frac{q_1}{\varepsilon_0} \Rightarrow$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r} = \underline{\underline{\frac{6 \times 10^{-9}}{4\pi\varepsilon_0 r^2}\hat{r} \ (SI)}}$$

S_2 : $\boxed{1 \leq r < r_e = 2\ \mathrm{m}}$

$$\phi_E = E \cdot S = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow E \cdot 4\pi r^2 = \frac{q_1 + q}{\varepsilon_0}$$

$$\text{sendo } \frac{q}{q_2} = \frac{V}{V_2} \Leftrightarrow q = q_2 \cdot \frac{\cancel{\frac{4}{3}}\pi(r^3 - r_i^3)}{\cancel{\frac{4}{3}}\pi(r_e^3 - r_i^3)} \Leftrightarrow$$

$$q = q_2 \cdot \frac{r^3 - 1^3}{2^3 - 1^3} = q_2 \cdot \frac{r^3 - 1}{7} \quad \text{donde}$$

$$\vec{E} = \frac{1}{4\pi r^2 \varepsilon_0} \cdot \left(q_1 + \frac{r^3 - 1}{7} q_2\right)\hat{r} \Leftrightarrow$$

$$\vec{E} = \frac{10^{-9}}{4\pi\varepsilon_0 r^2}\left[6 - \frac{4}{7}(r^3 - 1)\right]\hat{r} = \frac{10^{-9}}{14\pi\varepsilon_0}\cdot\left(\frac{23 - 2r^3}{r^2}\right)\hat{r} \ (N/C)$$

S_3 : $\boxed{r \geq 2}$

$$\phi_E = E\cdot S = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow E\cdot 4\pi r^2 = \frac{q_1 + q_2}{\varepsilon_0} \Leftrightarrow$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 + q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\times 10^{-9}}{r^2}\hat{r} \ (N/C)$$

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4. Num condutor as cargas eléctricas distribuem-se à superfície de tal forma que no seu interior o campo eléctrico se anula, donde :

S_1 : $\boxed{r < 1\,m}$ igual ao exercício anterior 3.

S_2 : $\boxed{1 \leq r < 2}$

$$\phi = ES = \frac{q_1 + q_{2i}}{\varepsilon_0}$$ sendo $E = 0$ resulta

$$q_{2i} = -q_1 = -6\,nC$$

S_3 : $\boxed{r \geq 2\,m}$ igual ao exercício 3.

temos ainda que $q_2 = q_{2e} + q_{2i} \Rightarrow$

$$q_{2e} = q_2 - q_{2i} = -4 + 6 = 2\,nC$$

assim, a carga $q_2 = -4\,nC$ distribui-se na superfície interior e na exterior nas seguintes quantidades: $q_{2i} = -6\,nC$ e $q_{2e} = 2\,nC$

5.1 $$\|\vec{E}\| = \left|\frac{\Delta V}{\Delta x}\right| = \frac{5\,kV}{8\,dm} = \frac{5\times10^{3}\,V}{8\times10^{-1}\,m} = 6250\ V/m$$

5.2 $$F_R = m\,a \Leftrightarrow e\,E = m_e\,a \Leftrightarrow$$

$$a = \frac{eE}{m_e} = \frac{1{,}6\times10^{-19}\times 6250}{9{,}1\times10^{-31}}$$

$$a \approx 1{,}1\times10^{15}\ m\,s^{-2}$$

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