1.1

$$\int_0^2\int_0^1 \left(3x^2y + y\,e^{3xy}\right) dx\,dy =$$

$$\int_0^2\int_0^1 \left(3x^2y + \frac{1}{3}\cdot 3y\,e^{3yx}\right) dx\,dy = \int_0^2 \left(x^3y + \frac{1}{3}e^{3yx}\right)\Big|_0^1 dy =$$

$$\int_0^2 \left(y + \frac{1}{3}e^{3y} - \frac{1}{3}\right) dy = \left(\frac{y^2}{2} + \frac{1}{9}e^{3y} - \frac{1}{3}y\right)\Big|_0^2 =$$

$$2 + \frac{1}{9}e^6 - \frac{2}{3} - \frac{1}{9} = \frac{11}{9} + \frac{1}{9}e^6$$

1.2

$$\int_{-1}^0\int_0^{\sqrt[3]{x}}\int_0^{xy} \frac{y}{x^3+2}\,dz\,dy\,dx = \int_{-1}^0\int_0^{\sqrt[3]{x}} \frac{y}{x^3+2}\,z\Big|_0^{xy} dy\,dx =$$

$$= \int_{-1}^0\int_0^{\sqrt[3]{x}} \frac{xy^2}{x^3+2}\,dy\,dx = \int_{-1}^0 \frac{x}{x^3+2}\cdot\frac{y^3}{3}\Big|_0^{\sqrt[3]{x}} dx =$$

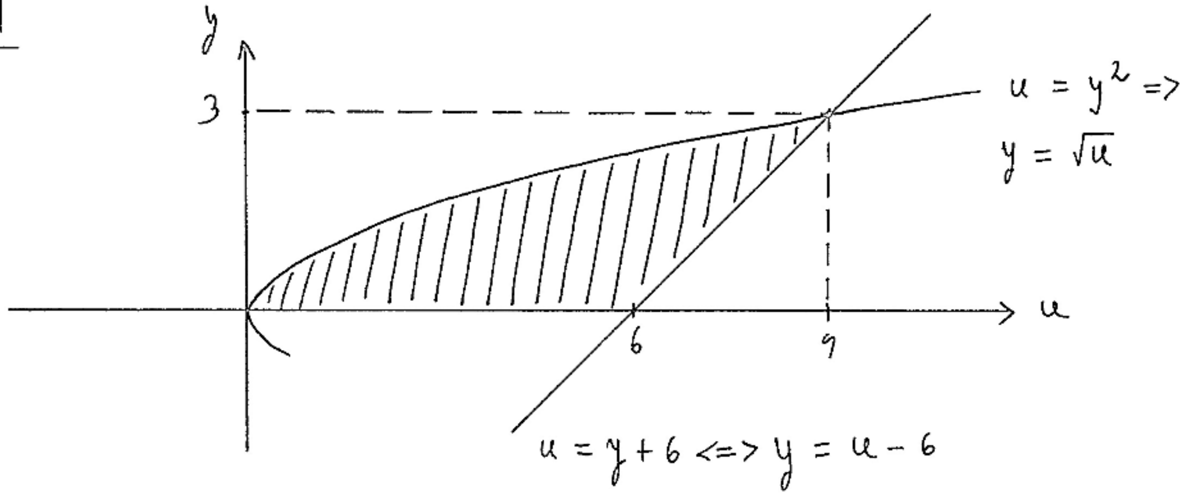
$$= \int_{-1}^0 \frac{x}{x^3+2}\cdot\frac{x}{3}\,dx = \frac{1}{3}\int_{-1}^0 \frac{x^2}{x^3+2}\,dx = \frac{1}{9}\int_{-1}^0 \frac{3x^2}{x^3+2}\,dx$$

$$= \frac{1}{9}\ln|x^3+2|\Big|_{-1}^0 = \frac{1}{9}(\ln 2 - \ln 1) = \frac{\ln 2}{9}$$

2.

$$\int_0^3 \int_{y^2}^{y+6} f(u,y)\, du\, dy$$

2.1



$$A = \int_0^3 \int_{y^2}^{y+6} du\, dy = \int_0^3 (y+6-y^2)\, dy = \left(\frac{y^2}{2} + 6y - \frac{y^3}{3}\right)\Bigg|_0^3 =$$

$$= \frac{9}{2} + 18 - 9 = -\frac{9}{2} + \frac{36}{2} = \underline{\underline{\frac{27}{2}}}$$

2.2

$$\int_0^3 \int_{y^2}^{y+6} f(u,y)\, du\, dy = \underline{\underline{\int_0^6 \int_0^{\sqrt{u}} f(u,y)\, dy\, du + \int_6^9 \int_{u-6}^{\sqrt{u}} f(u,y)\, dy\, du}}$$

2.3

$$\int_0^3 \int_{y^2}^{y+6} \frac{1}{(u+1)^2}\, du\, dy = \int_0^3 -\frac{1}{u+1}\Bigg|_{y^2}^{y+6} dy = \int_0^3 \left(-\frac{1}{y+7} + \frac{1}{y^2+1}\right) dy$$

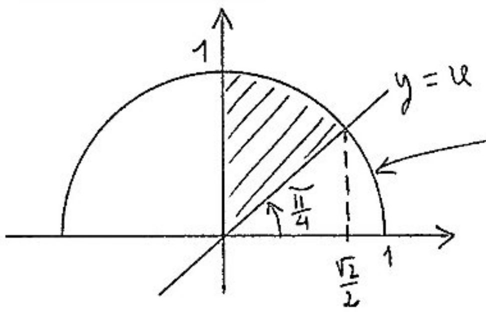
$$= \left(-\ln|y+7| + \operatorname{arctg} y\right)\Big|_0^3 = (-\ln 10 + \operatorname{arctg} 3) - (-\ln 7 + \operatorname{arctg} 0)$$

$$= \underline{\underline{\ln\frac{7}{10} + \operatorname{arctg} 3}}$$

3. $$\int_0^{\sqrt{2}/2}\int_u^{\sqrt{1-u^2}} \frac{u}{\sqrt{u^2+y^2}}\,dy\,du =$$

—————— // ——————

$$R = \left\{(u,y) \in \mathbb{R}^2 : 0 \le u \le \frac{\sqrt{2}}{2} \wedge u \le y \le \sqrt{1-u^2}\right\}$$



$$y = \sqrt{1-u^2} \Rightarrow u^2+y^2 = 1$$

$$\begin{cases} y = u \\ u^2+y^2=1 \end{cases} \quad \begin{cases} \overline{} \\ 2u^2 = 1 \end{cases} \quad \begin{cases} \overline{} \\ u = \pm\frac{\sqrt{2}}{2} \end{cases}$$

$$\left.\begin{array}{l} u = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \quad u^2+y^2 = r^2$$

$$dy\,du = r\,dr\,d\theta$$

$$R = \left\{(r,\theta) \in \mathbb{R}^2 : 0 \le r \le 1 \wedge \frac{\pi}{4} \le \theta \le \frac{\pi}{2}\right\}$$

—————— // ——————

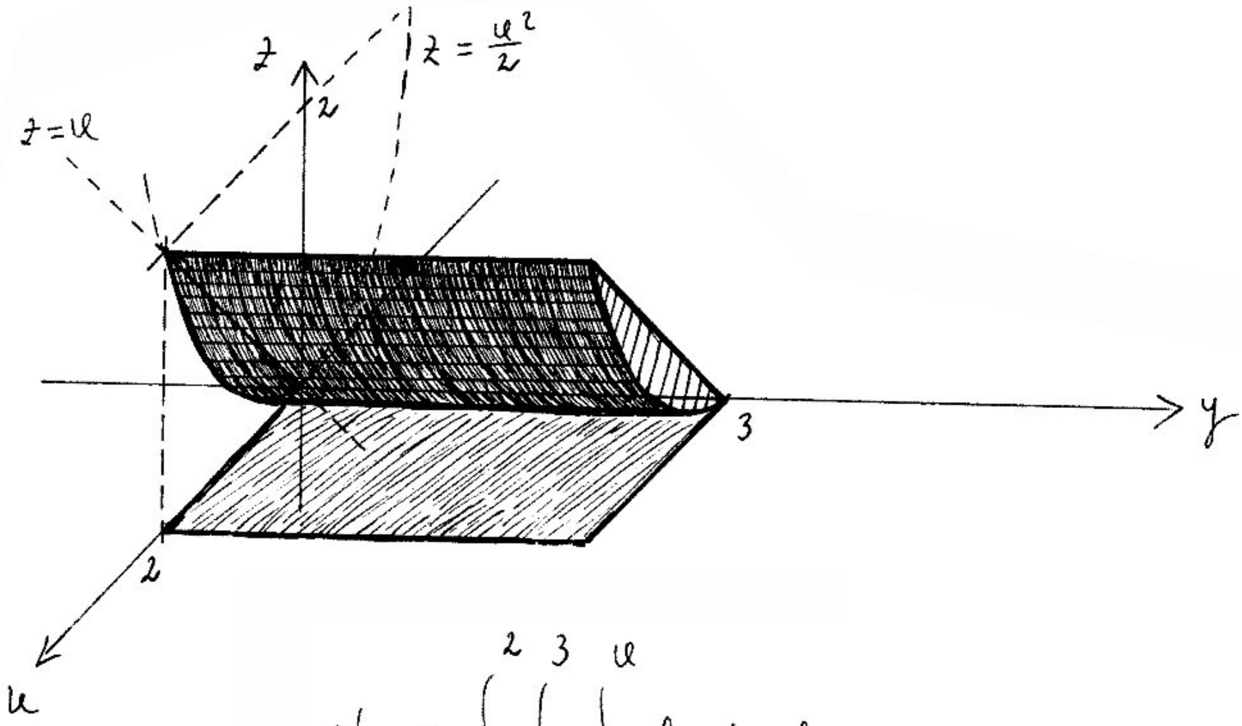
$$= \int_{\pi/4}^{\pi/2}\int_0^1 \frac{r\cos\theta}{\sqrt{r^2}}\,r\,dr\,d\theta = \int_{\pi/4}^{\pi/2}\int_0^1 r\cos\theta\,dr\,d\theta =$$

$$= \int_{\pi/4}^{\pi/2} \left.\frac{r^2}{2}\cos\theta\right|_0^1 d\theta = \int_{\pi/4}^{\pi/2} \frac{1}{2}\cos\theta\,d\theta = \left.\frac{1}{2}\sin\theta\right|_{\pi/4}^{\pi/2} =$$

$$= \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right) = \frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right) = \frac{2-\sqrt{2}}{4}$$

4.

$$W = \left\{ (u,y,z) \in \mathbb{R}^3 : \frac{u^2}{2} \leq z \leq u \wedge u > 0 \wedge 0 \leq y \leq 3 \right\}$$



$$V_W = \int_0^2 \int_0^3 \int_{\frac{u^2}{2}}^{u} dz\,dy\,du$$

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