actividade: *Resolução de Prova*
instituição: *Universidade Lusíada (Famalicão)*
disciplinas: *Matemática II*
tipologia: *Miniteste 1 - PL*
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1.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ -1 & 4 & -3 & 5 \\ 5 & -1 & 1 & 0 \end{array}\right] \underset{\substack{L_2 \to L_2 + L_1 \\ L_3 \to L_3 - 5L_1}}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 3 & -2 & 3 \\ 0 & 4 & -4 & 10 \end{array}\right] \underset{L_3 \to L_3/2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 3 & -2 & 3 \\ 0 & 2 & -2 & 5 \end{array}\right] \underset{\substack{L_1 \to 2L_1 + L_3 \\ L_2 \to L_2 - L_3}}{\sim} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 2 & -2 & 5 \end{array}\right] \underset{L_3 \to L_3 - 2L_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & 9 \end{array}\right] \underset{\substack{L_1 \to L_1/2 \\ L \to L /-2}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -9/2 \end{array}\right]$$

$$\therefore \begin{cases} x_1 = \frac{1}{2} \\ x_2 = -2 \\ x_3 = -9/2 \end{cases}$$

2.

2.1

$$\vec{AB} = B - A = (0,-2,0) - (3,0,1) = \langle -3,-2,-1 \rangle$$

$$\vec{BC} = C - B = (1,0,4) - (0,-2,0) = \langle 1,2,4 \rangle$$

$$\vec{\omega}_1 = \operatorname{proj}_{\vec{BC}} \vec{AB} = \frac{\vec{AB} \circ \vec{BC}}{\|\vec{BC}\|^2} \vec{BC} =$$

$$= \frac{\langle -3,-2,-1 \rangle \circ \langle 1,2,4 \rangle}{1^2 + 2^2 + 4^2} \langle 1,2,4 \rangle = \frac{-3-4-4}{21} \langle 1,2,4 \rangle$$

$$= -\frac{11}{21} \langle 1,2,4 \rangle = -\frac{11}{21}\hat{\imath} - \frac{22}{21}\hat{\jmath} - \frac{44}{21}\hat{k}$$

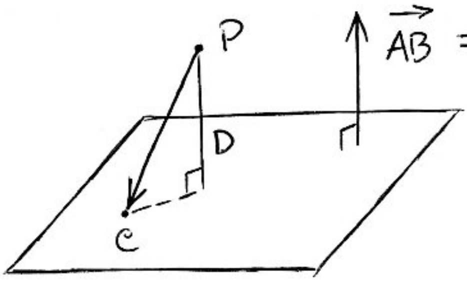
$$\vec{\omega}_2 = \vec{AB} - \vec{\omega}_1 =$$

$$= \left\langle -3, -2, -1 \right\rangle - \left\langle -\frac{11}{21}, -\frac{22}{21}, -\frac{44}{21} \right\rangle =$$

$$= \left\langle -3 + \frac{11}{21}, -2 + \frac{22}{21}, -1 + \frac{44}{21} \right\rangle =$$

$$= \left\langle -\frac{52}{21}, -\frac{20}{21}, \frac{23}{21} \right\rangle = -\frac{52}{21}\hat{i} - \frac{20}{21}\hat{j} + \frac{23}{21}\hat{k}$$

2.2



$$\vec{AB} = \vec{n} = \langle -3, -2, -1 \rangle$$

$$\vec{PC} = C - P = (1,0,4) - (1,2,3)$$

$$= \langle 0, -2, 1 \rangle$$

$$D = \frac{|\vec{PC} \circ \vec{n}|}{\|\vec{n}\|} = \frac{|\langle 0, -2, 1 \rangle \circ \langle -3, -2, -1 \rangle|}{\sqrt{3^2 + 2^2 + 1^2}} =$$

$$= \frac{4 - 1}{\sqrt{14}} = \frac{3}{\sqrt{14}} \approx 0{,}802$$

2.3

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & -1 \\ 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 2 & 4 \end{vmatrix}\hat{i} - \begin{vmatrix} -3 & -1 \\ 1 & 4 \end{vmatrix}\hat{j} + \begin{vmatrix} -3 & -2 \\ 1 & 2 \end{vmatrix}\hat{k}$$

$$= -6\hat{i} + 11\hat{j} - 4\hat{k}$$

$$\|\vec{AB} \times \vec{BC}\| = \sqrt{6^2 + 11^2 + 4^2} = \sqrt{173}$$

$$\therefore \hat{n} = \pm \frac{\vec{AB} \times \vec{BC}}{\|\vec{AB} \times \vec{BC}\|} = \pm \frac{-6\hat{i} + 11\hat{j} - 4\hat{k}}{\sqrt{173}} \cong$$

$$-0{,}456\hat{i} + 0{,}836\hat{j} - 0{,}3\hat{k}$$

2.4

$$\begin{cases} \cos\alpha = -\dfrac{6}{\sqrt{173}} \\ \cos\beta = \dfrac{11}{\sqrt{173}} \\ \cos\gamma = -\dfrac{4}{\sqrt{173}} \end{cases}$$

3.

3.1

$$\vec{v} \perp \vec{w} \iff \vec{v} \circ \vec{w} = 0 \iff$$

$$\langle 4, -1, 2 \rangle \circ \langle \alpha, 2, 1 \rangle = 0 \iff$$

$$4\alpha \cancel{-2} \cancel{+2} = 0 \iff 4\alpha = 0 \iff$$

$$\alpha = 0$$

3.2

$$A(1, -1, 2), \quad \vec{u} = 2\hat{j} + \hat{k} = \langle 0, 2, 1 \rangle$$

$$\begin{cases} x = 1 + 0t \\ y = -1 + 2t \\ z = 2 + t \end{cases}, t \in \mathbb{R} \iff \begin{cases} x = 1 \\ y = -1 + 2t \\ z = 2 + t \end{cases}, t \in \mathbb{R}$$

3.3

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 4 & -1 & 2 \end{vmatrix} = (4 - (-1))\hat{i} - (0 - 4)\hat{j} + (0 - 8)\hat{k}$$

$$= 5\hat{i} + 4\hat{j} - 8\hat{k}$$

$$A = \|\vec{u} \times \vec{v}\| = \sqrt{5^2 + 4^2 + 8^2} = \sqrt{105} \approx 10{,}25$$

3.4

$$\rho = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\theta = \operatorname{arctg} \frac{-1}{1} + 360° = 315° \quad \therefore \quad A(\sqrt{2}, 315°, 2)$$

$$z = 2$$

4.

4.1 $D_f = \{(x,y) \in \mathbb{R}^2 : -9x^2 - y^2 + 9 \geqslant 0\}$

$$-9x^2 - y^2 + 9 \geqslant 0 \Leftrightarrow$$
$$9x^2 + y^2 \leqslant 9 \Leftrightarrow$$
$$x^2 + \frac{y^2}{9} \leqslant 1 \Leftrightarrow x^2 + \frac{y^2}{3^2} \leqslant 1$$

Elipse e interior de centro na origem e semi-eixos 1 e 3, horizontal e vertical, respetivamente.

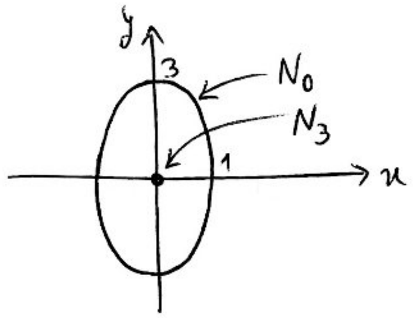
4.2 $N_c = \{(x,y) \in \mathbb{R}^2 : \sqrt{-9x^2 - y^2 + 9} = c\}$

$$\sqrt{-9x^2 - y^2 + 9} = c \Rightarrow -9x^2 - y^2 + 9 = c^2 \wedge c \geqslant 0$$
$$9x^2 + y^2 - 9 = -c^2 \wedge c \geqslant 0 \Leftrightarrow$$
$$9x^2 + y^2 = 9 - c^2 \wedge 0 \leqslant c \leqslant 3$$

As curvas de nível são elipses com exceção da curva de nível 3, que é um ponto:

$N_3 = \{(0,0)\}$ $\quad 9x^2 + y^2 = 0 \Rightarrow x = 0 \wedge y = 0$

$N_0 = \{(x,y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{3^2} = 1\}$



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