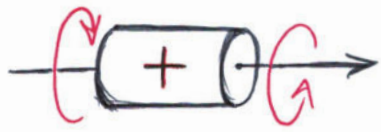
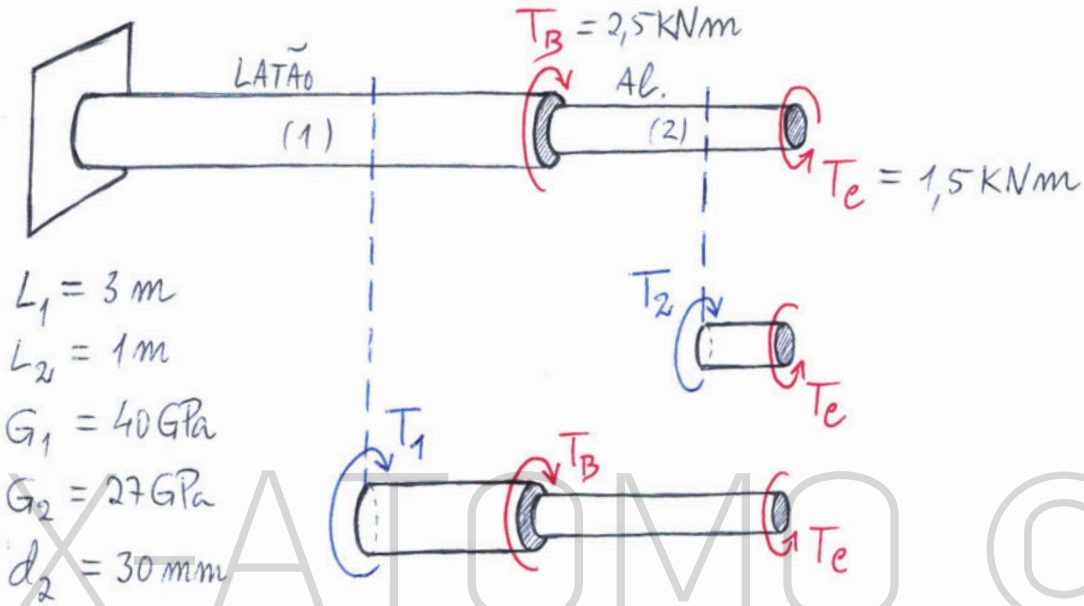
18-12-2018 RM UL

1.1 $\tau_{adm} = 50\ MPa$

$\phi_{AC} = 0{,}02\ rad$

CONVENÇÃO DE SINAL:





$-T_2 + T_e = 0 \Rightarrow T_2 = T_e = 1{,}5\ kNm$ (+)

$-T_1 - T_B + T_e = 0 \Rightarrow T_1 = T_e - T_B = -1{,}0\ kNm$ (−)

$$\phi_{AC} = \phi_1 + \phi_2 = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} =$$

$$= \frac{-1\times10^{3}\times 3}{\frac{\pi}{2}\times e_1^{4}\times 40\times10^{9}} + \frac{1{,}5\times10^{3}\times 1}{\frac{\pi}{2}\times 0{,}015^{-4}\times 27\times10^{9}} =$$

$$= -\frac{4{,}7746\times10^{-8}}{e_1^{4}} + 0{,}6986.$$

$$\begin{cases} \tau_{máx,AB} \leq \tau_{adm} \\ |\phi_{AC}| \leq 0{,}02 \text{ rad} \end{cases} \quad \begin{cases} \dfrac{|T_1|\, c_1}{J_1} \leq \tau_{adm} \\ -0{,}02 \leq \phi_{AC} \leq 0{,}02 \end{cases} \Leftrightarrow$$

$$\begin{cases} \dfrac{|T_1|\, c_1}{\frac{\pi}{2} c_1^4} \leq \tau_{adm} \\ -0{,}02 \leq -\dfrac{4{,}7746 \times 10^{-8}}{c_1^4} + 0{,}6986 \leq 0{,}02 \end{cases} \Leftrightarrow$$

$$\begin{cases} c_1 \geq \sqrt[3]{\dfrac{2|T_1|}{\pi \tau_{adm}}} \\ -0{,}7186 \leq -\dfrac{4{,}7746 \times 10^{-8}}{c_1^4} \leq -0{,}6786 \end{cases} \Leftrightarrow$$

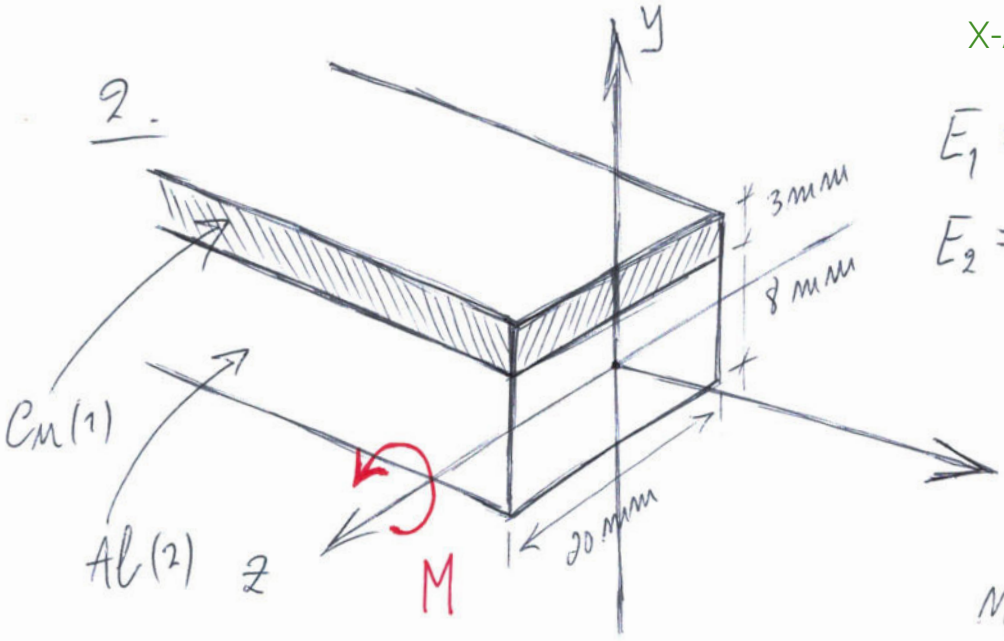
$$\begin{cases} c_1 \geq \sqrt[3]{\dfrac{2 \times 1 \times 10^{3}}{\pi \times 50 \times 10^{6}}} \\ 0{,}6786 \leq \dfrac{4{,}7746 \times 10^{-8}}{c_1^4} \leq 0{,}7186 \end{cases} \Leftrightarrow$$

$$\begin{cases} c_1 \geq 23{,}35 \text{ mm} \\ 16{,}1 \text{ mm} \leq c_1 \leq 16{,}3 \text{ mm} \end{cases}$$

logo, não há nenhum diâmetro que satisfaça, simultaneamente, as duas condições.

1.2. $\phi_{AC} = -\dfrac{4{,}7746 \times 10^{-8}}{0{,}050^4} + 0{,}6986 = 0{,}691 \text{ rad} = 39{,}6°$

2.

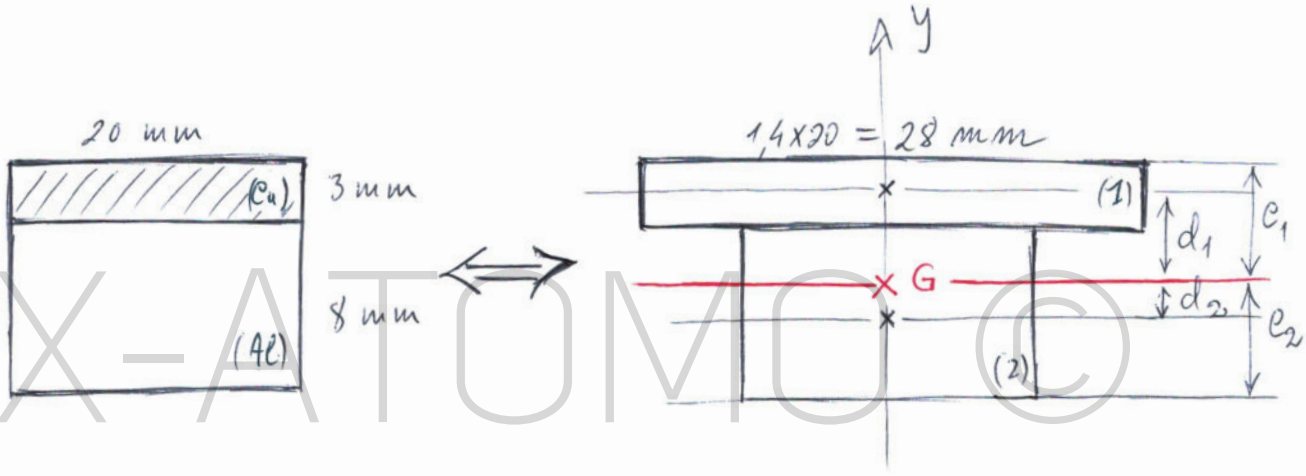


$E_1 = E_{Cu} = 105\ GPa$

$E_2 = E_{Al} = 75\ GPa$

$M = 40\ kN \cdot m$

$$n = \frac{E_{Cu}}{E_{Al}} = 1{,}4$$



$$y_G = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{28 \times 3 \times 9{,}5 + 20 \times 8 \times 4}{28 \times 3 + 20 \times 8} = 5{,}89\ mm$$

$$I = \left(I_{G1} + A_1 d_1^2\right) + \left(I_{G2} + A_2 d_2^2\right) =$$

$$= \frac{28 \times 3^3}{12} + 28 \times 3 \times (9{,}5 - 5{,}89)^2 + \frac{20 \times 8^3}{12} + 20 \times 8 (5{,}89 - 4)^2$$

$$= 2582{,}6\ mm^4 = 2{,}5826 \times 10^3\ mm^4$$

$$= 2{,}5826 \times 10^{-9}\ m^4$$

$$\sigma_{máx,\,comp} = n\frac{Mc_1}{I} = 1{,}4 \times \frac{40\times10^{3} \times \left|11-5{,}89\right|\times10^{-3}}{2{,}5826\times10^{-9}}$$

$$= 110{,}8\times10^{9}\ Pa = 110{,}8\ GPa$$

$$\sigma_{máx,\,TRAÇÃO} = \frac{Mc_2}{I} = \frac{40\times10^{3}\times5{,}89\times10^{-3}}{2{,}5826\times10^{-9}}$$

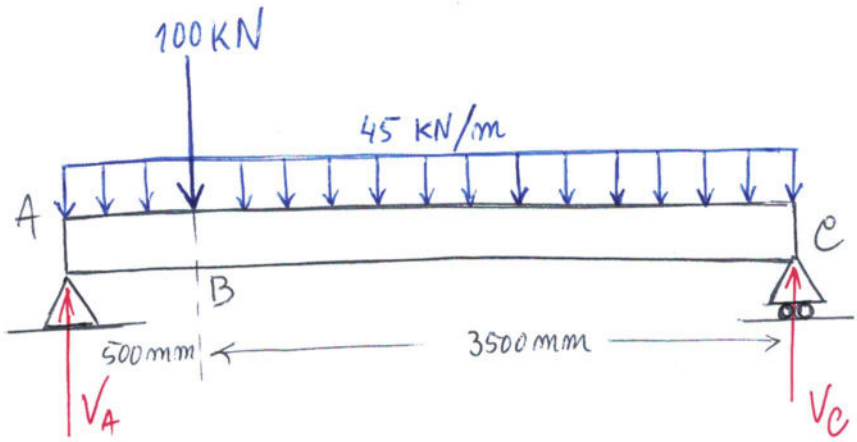
$$= 91{,}2\times10^{9}\ Pa = 91{,}2\ GPa$$

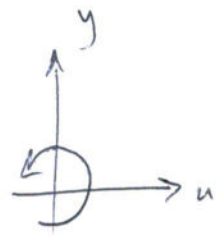
2.2

$$\rho = \frac{E_2\, I}{M} = \frac{75\times10^{9}\times2{,}5826\times10^{-9}}{40\times10^{3}}$$

$$\rho = 4{,}8\times10^{-3}\ m = 4{,}8\ mm\ !!!$$

3.



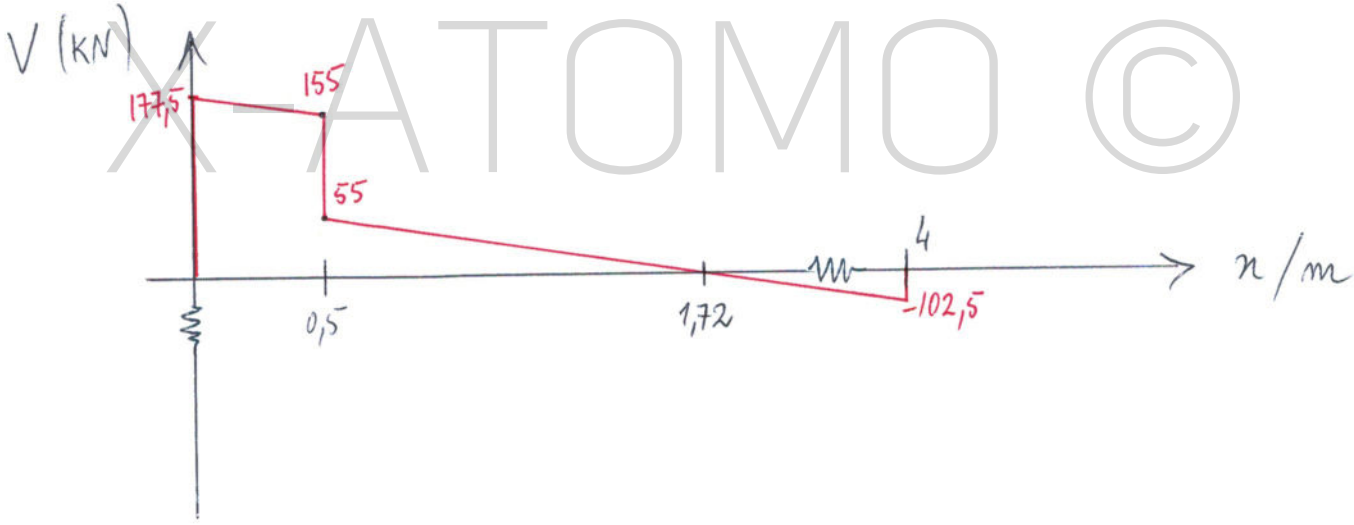


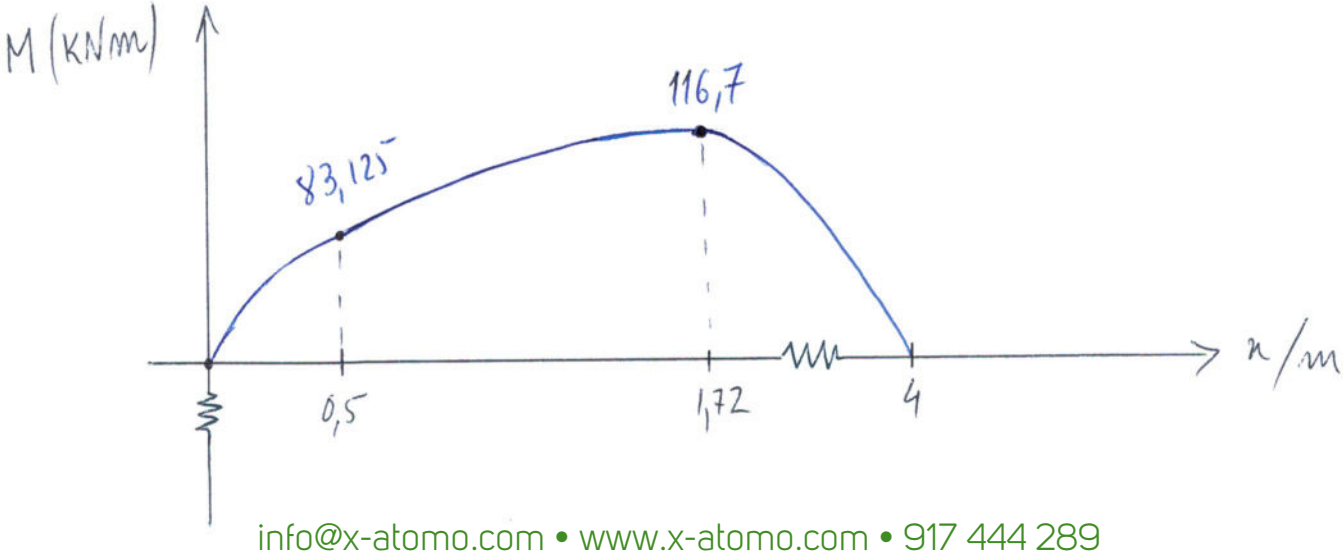
$$\sum M_A = 0 \iff 4000\,V_C - 500 \times 100 - 2000 \times 45 \times 4 = 0$$

$$\iff V_C = \underline{\underline{102,5\ kN}}$$

$$\sum F_y = 0 \iff V_A - 100 - 45 \times 4 + 102,5 = 0$$

$$\iff V_A = \underline{\underline{177,5\ kN}}$$





$$\frac{x}{102,5} = \frac{3,5}{157,5} \Rightarrow x = 2,28 \text{ m}$$

CÁLCULOS

$0 \leq x < 0,5$ m

$V_A = 177,5$ kN

$$V(x) = V_A + \int_0^x -45\,dx = 177,5 - 45x \quad (kN)$$

$$V(0,5^-) = 177,5 - 45 \times 0,5 = 155 \text{ kN}$$

$0,5 \leq x < 4$ m

$$V(x) = F(0,5) + V(0,5^-) + \int_{0,5}^x -45\,dx = -100 + 155 - 45(x - 0,5)$$

$$= -45x + 77,5 \text{ kN}$$

$$V(x) = 0 \Leftrightarrow -45x + 77,5 = 0 \Leftrightarrow x = \frac{77,5}{45} = 1,72 \text{ m}$$

$$V(4^-) = -45 \times 4 + 77,5 = -102,5 \text{ kN}$$

$$V(4) = -102,5 + V_e = 0$$

$0 \leq x < 0,5$ m

$$M(x) = \int_0^x (177,5 - 45x)\,dx = 177,5x - 22,5x^2 \quad (kNm)$$

$$M(0,5^-) = 177,5 \times 0,5 - 22,5 \times 0,5^2 = 83,125 \text{ kNm}$$

$0,5 \leq x < 4$ m

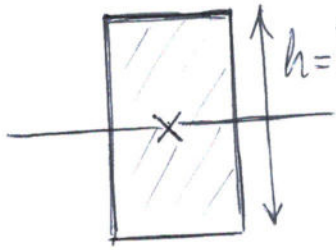
$$M(x) = 83,125 + \int_{0,5}^x (-45x + 77,5)\,dx = 83,125 - 22,5(x^2 - 0,5^2) + 77,5(x - 0,5)$$

$M(x) = -22{,}5x^2 + 77{,}5x + 50 \quad (kNm)$

$M(1{,}72) = 116{,}7\ kNm = M_{máx}$

$M(4) = 0$

3.2.



$b = 130mm$

$I = \frac{bh^3}{12}$

$\sigma_{adm} = 16\ MPa$

$\sigma_{máx} < \sigma_{adm} \Leftrightarrow \frac{M \cdot c}{I} < \sigma_{adm} \Leftrightarrow$

$\frac{M_{máx} \cdot h/2}{bh^3/12} < \sigma_{adm} \Leftrightarrow \frac{6M_{máx}}{bh^2} < \sigma_{adm} \Leftrightarrow$

$h^2 > \frac{6M_{máx}}{\sigma_{adm} \cdot b} \Rightarrow h > \sqrt{\frac{6M_{máx}}{\sigma_{adm} \cdot b}}$

$h > \sqrt{\frac{6 \times 116{,}7 \times 10^3}{16 \times 10^6 \times 0{,}130}} = 0{,}580m = 580\ mm$